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**AMPLITUDE RELATIONS FOR  $B$  DECAYS INVOLVING  $\eta$  and  $\eta'$ <sup>1</sup>***Amol S. Dighe**Enrico Fermi Institute and Department of Physics  
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A class of amplitude relations for decays of  $B$  mesons is discussed. Processes involving  $\eta$  and  $\eta'$  in the final state are shown to provide useful information about weak phases in some cases even in the presence of octet-singlet mixing in these states. Some of the relations are unaffected by first-order SU(3) breaking.

The decays of  $B$  mesons can provide unique insights into the details of the weak interactions. Among the most eagerly sought of these is a confirmation that phases in the elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix [1] are responsible for the observed violation of CP invariance [2] in decays of neutral kaons.

In the present report we describe a class of  $B$  decays which can shed light on CKM phases through the construction of relations satisfied by complex decay amplitudes. Previous results [3, 4, 5] involving the decays  $B \rightarrow \pi\pi$ ,  $B \rightarrow \pi K$ , and  $B \rightarrow K\bar{K}$  have indicated the possibility of isolating CKM phases from the effects of strong interactions using such relations, but the constructions were complicated by the presence of higher-order electroweak (“penguin”) contributions [6, 7, 8, 9]. The inclusion of decays involving  $\eta$  mesons [10] provides enough information to avoid this complication as long as  $\eta$  is regarded as a flavor octet member. Our discussion, involving  $\eta$  and  $\eta'$  mesons, takes account of the observed octet-singlet mixing in the  $\eta$  and  $\eta'$  [11]. A more complete discussion is presented in Ref. [12].

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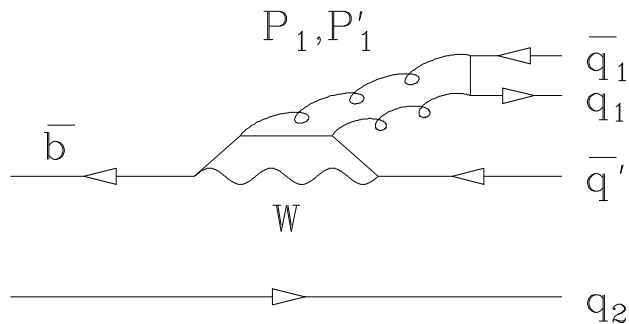


Figure 1: Graph contributing to  $B$  decays involving one or two flavor SU(3) singlet pseudoscalar mesons. The coiled lines denote a color singlet exchange due to two or more gluons. Here  $q_1, q_2 = u, d, s$  while  $q' = d, s$ .

Our method employs flavor SU(3) symmetry [13, 14, 15, 16] in order to relate decays involving pairs of light pseudoscalar mesons in the final state. We present results for the case, very close to experiment [11, 15], in which  $\eta = (s\bar{s} - u\bar{u} - d\bar{d})/\sqrt{3}$  and  $\eta' = (2s\bar{s} + u\bar{u} + d\bar{d})/\sqrt{6}$ . We retain only those amplitudes in which the spectator quark (the light quark accompanying the  $b$  in the initial meson) does not enter into the decay Hamiltonian. These amplitudes are expected to be the dominant ones; others are expected to be suppressed by a power of  $f_B/m_B$  or  $f_{B_s}/m_{B_s}$ , where  $f_B \simeq 180$  MeV or  $f_{B_s} \simeq 225$  MeV are decay constants. Most of our results are unaffected by first-order SU(3) breaking, for which a partial set of tests has been presented elsewhere [17].

In the present approximation [12, 18] there are four independent amplitudes: a “tree” contribution  $t$ , a “color-suppressed” contribution  $c$ , a “penguin” contribution  $p$ , and a “singlet penguin” contribution  $p_1$ , in which a color-singlet  $q\bar{q}$  pair produced by two or more gluons (see Fig. 1) or by a  $Z$  or  $\gamma$  forms an SU(3) singlet state. These amplitudes contain both the leading-order and electroweak penguin contributions:

$$\begin{aligned} t &\equiv T + (c_u - c_d)P_{EW}^C, & c &\equiv C + (c_u - c_d)P_{EW}, \\ p &\equiv P + c_dP_{EW}^C, & p_1 &\equiv P_1 + c_dP_{EW}, \end{aligned} \quad (1)$$

where the capital letters denote the leading-order contributions (as defined, for example, in Ref. [3]), and  $P_{EW}$  and  $P_{EW}^C$  are color-favored and color-suppressed electroweak penguin amplitudes, as defined in Ref. [9]. Here  $c_u$  and  $c_d$  depend on the structure of the electroweak penguin; amplitudes were redefined in Ref. [9] such that  $c_u = 2/3$  and  $c_d = -1/3$ . As shown earlier [9], the electroweak penguin contributions may be incorporated without affecting SU(3) analyses as long as one does not attempt to relate  $\Delta S = 0$  and  $|\Delta S| = 1$  processes. In what follows we shall denote the  $\Delta S = 0$  processes by unprimed quantities and the  $|\Delta S| = 1$  processes by primed quantities.

The results for decays involving  $\eta$  and  $\eta'$  are shown in Tables 1 and 2. Decompositions of amplitudes for  $\pi\pi$ ,  $\pi K$ , and  $KK$  decays in terms of the three contributions  $t(t')$ ,  $c(c')$ , and  $p(p')$  have been given elsewhere [3]. Our notation [18] is equivalent to that of Ref. [12].

Table 1: Decomposition of  $B \rightarrow PP$  amplitudes involving physical  $\eta$  and  $\eta'$  (as defined in the text) for  $\Delta C = \Delta S = 0$  transitions in terms of graphical contributions.

	Final state	$t$	$c$	$p$	$p_1$
$B^+ \rightarrow$	$\pi^+ \eta$	$-1/\sqrt{3}$	$-1/\sqrt{3}$	$-2/\sqrt{3}$	$-1/\sqrt{3}$
	$\pi^+ \eta'$	$1/\sqrt{6}$	$1/\sqrt{6}$	$2/\sqrt{6}$	$2\sqrt{2/3}$
$B^0 \rightarrow$	$\pi^0 \eta$	0	0	$-2/\sqrt{6}$	$-1/\sqrt{6}$
	$\pi^0 \eta'$	0	0	$1/\sqrt{3}$	$2/\sqrt{3}$
	$\eta\eta$	0	$\sqrt{2}/3$	$\sqrt{2}/3$	$\sqrt{2}/3$
	$\eta\eta'$	0	$-\sqrt{2}/3$	$-\sqrt{2}/3$	$-5/3\sqrt{2}$
	$\eta'\eta'$	0	$\sqrt{2}/6$	$\sqrt{2}/6$	$2\sqrt{2}/3$
$B_s \rightarrow$	$\eta \bar{K}^0$	0	$-1/\sqrt{3}$	0	$-1/\sqrt{3}$
	$\eta' \bar{K}^0$	0	$1/\sqrt{6}$	$3/\sqrt{6}$	$2\sqrt{2/3}$

Table 2: Decomposition of  $B \rightarrow PP$  amplitudes for  $\Delta C = 0$ ,  $|\Delta S| = 1$  transitions involving physical  $\eta$  and  $\eta'$  (as defined in the text) in terms of graphical contributions.

	Final state	$t'$	$c'$	$p'$	$p'_1$
$B^+ \rightarrow$	$\eta K^+$	$-1/\sqrt{3}$	$-1/\sqrt{3}$	0	$-1/\sqrt{3}$
	$\eta' K^+$	$1/\sqrt{6}$	$1/\sqrt{6}$	$3/\sqrt{6}$	$2\sqrt{2/3}$
$B^0 \rightarrow$	$\eta K^0$	0	$-1/\sqrt{3}$	0	$-1/\sqrt{3}$
	$\eta' K^0$	0	$1/\sqrt{6}$	$3/\sqrt{6}$	$2\sqrt{2/3}$
$B_s \rightarrow$	$\pi^0 \eta$	0	$-1/\sqrt{6}$	0	0
	$\pi^0 \eta'$	0	$-1/\sqrt{3}$	0	0
	$\eta\eta$	0	$-\sqrt{2}/3$	$\sqrt{2}/3$	$-\sqrt{2}/3$
	$\eta\eta'$	0	$-1/3\sqrt{2}$	$2\sqrt{2}/3$	$\sqrt{2}/3$
	$\eta'\eta'$	0	$\sqrt{2}/3$	$2\sqrt{2}/3$	$4\sqrt{2}/3$

Several amplitude sum rules involving  $\eta$  and  $\eta'$  are implied by the Tables. A more complete list (including some  $B_s$  decays omitted here) may be found in Ref. [12]. We label those which are unaffected by first-order SU(3) breaking [17] by an asterisk.

$\Delta S = 0$  decays:

\*1. A relation involving only pions and  $\eta$ 's is

$$\begin{aligned} & \sqrt{2}A(B^+ \rightarrow \pi^+\pi^0) - \sqrt{3}A(B^+ \rightarrow \pi^+\eta) + \sqrt{6}A(B^0 \rightarrow \pi^0\eta) \\ & = -(t+c) + (t+c+2p+p_1) - (2p+p_1) = 0 \quad . \end{aligned} \quad (2)$$

Since the  $B^0 \rightarrow \pi^0\eta$  process involves only  $p$  and  $p_1$  (including electroweak penguin contributions), its weak phase is approximately  $\text{Arg } V_{td} = -\beta$ , if we neglect corrections [19] due to quarks other than the top quark. The weak phase of  $t+c$  is approximately  $\text{Arg } V_{ub}^* = \gamma$ , if one neglects electroweak penguin contributions which should be small in this case [9]. The relative weak phase of the two amplitudes is then  $\gamma + \beta = \pi - \alpha$ , where  $\alpha$ ,  $\beta$ , and  $\gamma$  are the three angles of the unitarity triangle. By comparing the triangle (2) with the corresponding one for the charge-conjugate processes, one can then measure not only  $\alpha$ , but also the strong phase difference between the  $t+c$  and  $2p+p_1$  terms.

The charged  $B$  decays are “self-tagging.” The  $B^0 \rightarrow \pi^0\eta$  and  $\bar{B}^0 \rightarrow \pi^0\eta$  rates are expected to be equal since the amplitudes  $p$  and  $p_1$  should have equal weak phases. Thus the only shortcoming of this triangle relation is the expected smallness of the  $B^0 \rightarrow \pi^0\eta$  decay rate, which we estimate (using  $|p| = \mathcal{O}(1/5)|t|$  and neglecting the unknown  $|p_1|$  contribution) to be less than  $10^{-6}$ . The amplitude triangles should have two long sides and a short one.

\*2. A relation involving only  $\pi^0$ 's and  $\eta$ 's is

$$\begin{aligned} & \sqrt{2}A(B^0 \rightarrow \pi^0\pi^0) + \sqrt{6}A(B^0 \rightarrow \pi^0\eta) + \frac{3}{\sqrt{2}}A(B^0 \rightarrow \eta\eta) \\ & = (p-c) - (2p+p_1) + (c+p+p_1) = 0 \quad . \end{aligned} \quad (3)$$

In this case, in contrast to the previous one, we have to identify the flavor at time of production of the decaying neutral  $B$ , since the rates for the  $\pi^0\pi^0$  and  $\eta\eta$  decays are not necessarily expected to be equal for  $B^0$  and  $\bar{B}^0$  decays. Since all the amplitudes are either penguins or color-suppressed, we expect all sides of the amplitude triangles to be small, corresponding to branching ratios of  $10^{-6}$  or less.

\*3. In the absence of the  $p_1$  contribution, we would have

$$A(B^+ \rightarrow \pi^+\eta) = -\sqrt{2}A(B^+ \rightarrow \pi^+\eta') = -(t+c+p)/\sqrt{3} \quad , \quad (4)$$

since the  $t$ ,  $c$ , and  $p$  contributions to these processes probe only the nonstrange quark content of the  $\eta$  and  $\eta'$ . Similarly, if  $p_1$  is neglected, we have

$$A(B^0 \rightarrow \pi^0\eta) = -\sqrt{2}A(B^0 \rightarrow \pi^0\eta') = -2p/\sqrt{6} \quad . \quad (5)$$

The degree of violation of these relations thus can be regarded as one measure of the importance of the  $p_1$  term. The relation for neutral  $B$  decays is probably more sensitive to  $p_1$  since it lacks the dominant  $t$  contribution. The  $p_1$  term receives contributions from both  $P_1$ , illustrated in Fig. 1, in which the flavor singlet  $q_1\bar{q}_1$  pair is connected to the rest of the diagram with at least two gluons, and the electroweak penguin term  $P_{EW}$ , as indicated in the last of Eqs. (1). It is quite likely that each of these is smaller than the  $p$  term, which in turn is likely to be  $\mathcal{O}(1/5)$  of the dominant  $t$  term.

4. The following amplitudes are expected to have approximately the same magnitudes for the  $B$  and  $\bar{B}$  decay processes, since they have a common weak phase in the limit of  $t$  dominance of  $p$  and  $p_1$ :

$$\begin{aligned} 3A(B^+ \rightarrow K^+ \bar{K}^0) + 2\sqrt{6}A(B^0 \rightarrow \pi^0 \eta) + \sqrt{3}A(B^0 \rightarrow \pi^0 \eta') \\ = 3p - 2(2p + p_1) + (p + 2p_1) = 0 \quad . \end{aligned} \quad (6)$$

Thus, in this approximation, one does not expect to see CP violation in any of the rates for these processes, and flavor tagging is unnecessary. The shape of the amplitude triangle provides information on the relative magnitudes and phases of  $p$  and  $p_1$ . The above relation is affected by first-order SU(3) breaking because it involves creation from the vacuum of both nonstrange and strange quark pairs.

\*5. There is an amplitude triangle involving decays with  $\eta$  and  $\eta'$ :

$$\begin{aligned} A(B^0 \rightarrow \eta\eta) + 2A(B^0 \rightarrow \eta\eta') + 2A(B^0 \rightarrow \eta'\eta') \\ = (\sqrt{2}/3)(c + p + p_1) - (\sqrt{2}/3)(2c + 2p + 5p_1) + (\sqrt{2}/3)(c + p + 4p_1) = 0 \quad , \end{aligned} \quad (7)$$

from which we can learn the relative weak and strong phases of  $c + p$  and  $p_1$ . Since we learn  $p_1/p$  from the previous construction, we can then learn  $c/p$ . Since electroweak penguins probably contribute in a non-negligible manner to  $c$ , this information may be of limited use in determining weak phases.

$|\Delta S| = 1$  decays:

1. A quadrangle relation [18] for charged  $B$  decays generalizes the triangle relation of Ref. [10]:

$$\begin{aligned} 2A(B^+ \rightarrow \pi^+ K^0) + \sqrt{2}A(B^+ \rightarrow \pi^0 K^+) - (4/\sqrt{3})A(B^+ \rightarrow \eta K^+) - (\sqrt{6}/3)A(B^+ \rightarrow \eta' K^+) \\ = 2p' - (t' + c' + p') + (4/3)(t' + c' + p'_1) - (1/3)(t' + c' + 3p' + 4p'_1) = 0 \quad . \end{aligned} \quad (8)$$

It is possible to inscribe in the quadrangle a triangle composed of linear combinations of  $p'$  and  $p'_1$ , whose shape does not change under charge conjugation. Thus, by studying the processes and their charge conjugates, one can construct rigid quadrangles (up to discrete ambiguities), from which – in the manner of Refs. [9] and [10], making use of the rate for  $B^+ \rightarrow \pi^+ \pi^0$  – one can learn the weak phase  $\gamma$  [18]. The absence of the term  $p'$  in the amplitude for  $B^+ \rightarrow \eta K^+$  (which arises as a result of cancellation of nonstrange and strange quark contributions for the particular mixture of octet and singlet assumed

here) suggests that this may be the process with the smallest branching ratio of the four. This relation is affected by first-order SU(3) breaking for the same reason as in Eq. (6).

2. A triangle relation satisfied by the neutral  $B$  decays is affected by first-order SU(3) breaking for the same reason as the previous result:

$$\begin{aligned} 3\sqrt{2}A(B^0 \rightarrow \pi^0 K^0) - 4\sqrt{3}A(B^0 \rightarrow \eta K^0) - \sqrt{6}A(B^0 \rightarrow \eta' K^0) \\ = 3(p' - c') + 4(c' + p'_1) - (c' + 3p' + 4p'_1) = 0 \quad . \end{aligned} \quad (9)$$

Here one learns  $c'$  relative to  $p'$  or  $p'_1$ . This information may not be so useful for the study of weak phases, but it can serve to check specific models.

\*3. The ratio of the rates for  $B_s \rightarrow \pi^0 \eta$  and  $B_s \rightarrow \pi^0 \eta'$  can check the relative strange quark content of the two states, since these amplitudes both depend only on  $c'$ . For the mixing adopted here, we predict

$$A(B_s \rightarrow \pi^0 \eta') = \sqrt{2}A(B_s \rightarrow \pi^0 \eta) \quad (10)$$

The  $\pi^0 \eta$  mode was suggested as one way of specifying the shape of the isospin quadrangle in  $B \rightarrow \pi K$  decays [9], avoiding the problems associated with electroweak penguins [7] that arose in an earlier construction [3, 4].

A number of additional  $|\Delta S| = 1$  decays involving  $B_s$  and  $\eta$  or  $\eta'$  are noted in Ref. [12].

To summarize, several interesting amplitude relations involving decays of  $B$  mesons with  $\eta$  and/or  $\eta'$  in the final state can shed light on weak phases and on relative strong phases and magnitudes of different contributions to amplitudes. Foremost among these are a triangle relation satisfied by the amplitudes for  $B^+ \rightarrow \pi^+ \pi^0$ ,  $B^+ \rightarrow \pi^+ \eta$ , and  $B^0 \rightarrow \pi^0 \eta$ , all unaffected by first-order SU(3) breaking, and a quadrangle relation [18] satisfied by the amplitudes for  $B^+ \rightarrow \pi^+ K^0$ ,  $\pi^0 K^+$ ,  $\eta K^+$ , and  $\eta' K^+$ . The detection of these modes poses an interesting instrumental problem well-matched to the capabilities of detectors at present or planned  $e^+e^-$  colliders.

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